# Data Science

## Lecture 9-1: Image Data Processing

## (Image Filtering)

University of Amsterdam

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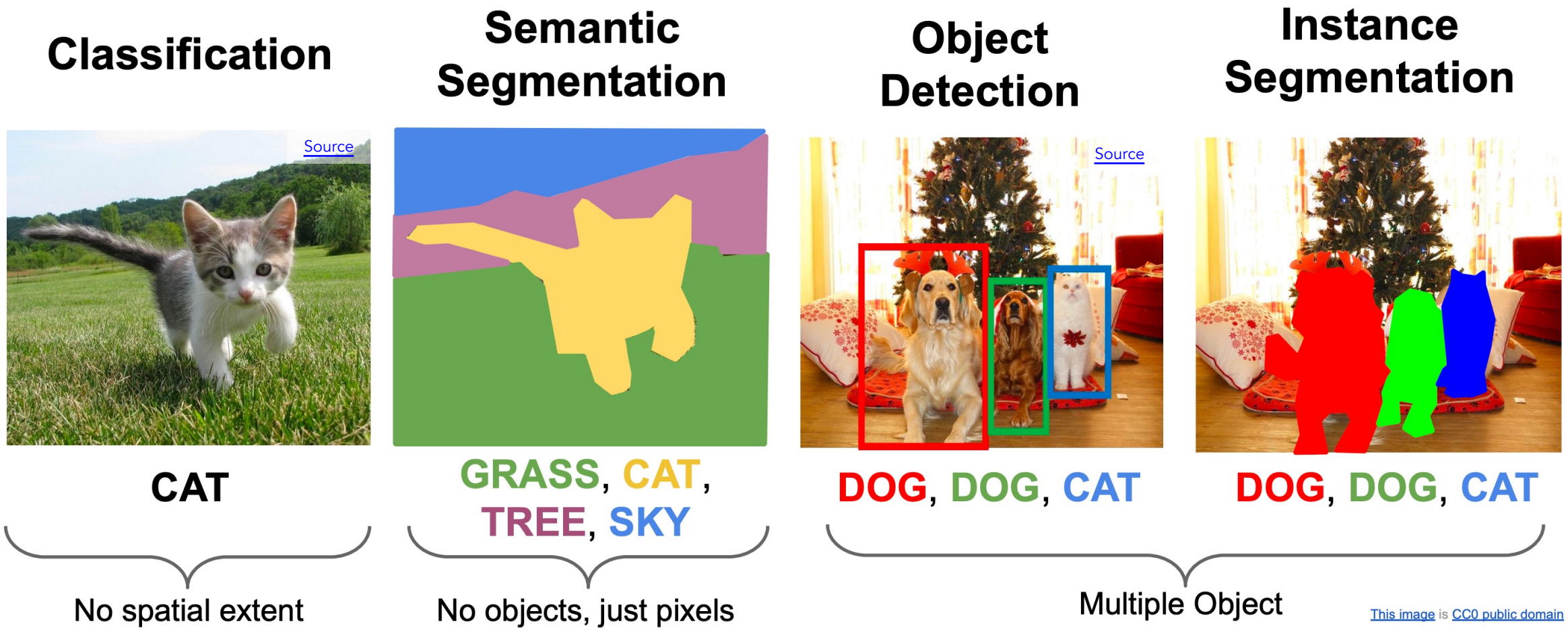
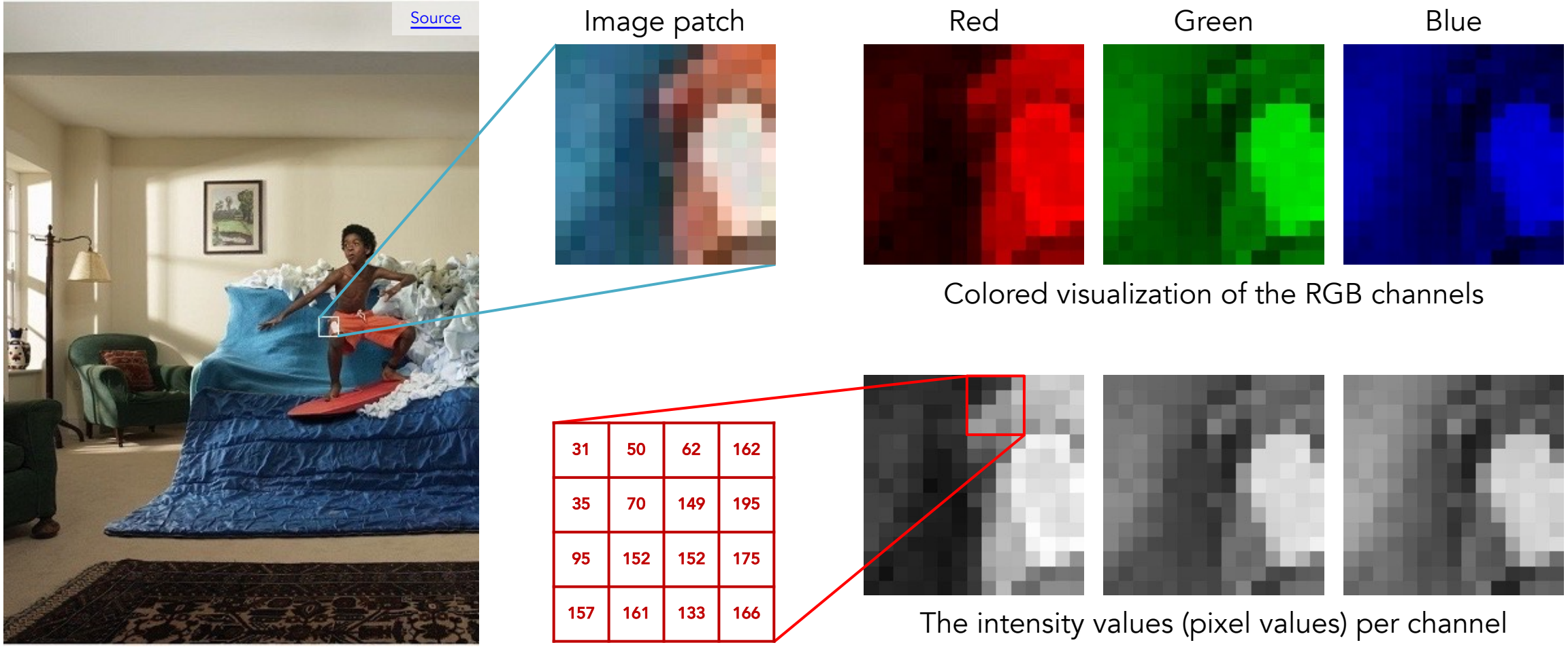
This lecture covers basic concepts of image filtering and feature extraction.

There are many different types of tasks in Computer Vision. This lecture will only cover the image/video classification, which gives only one label to an image/video.



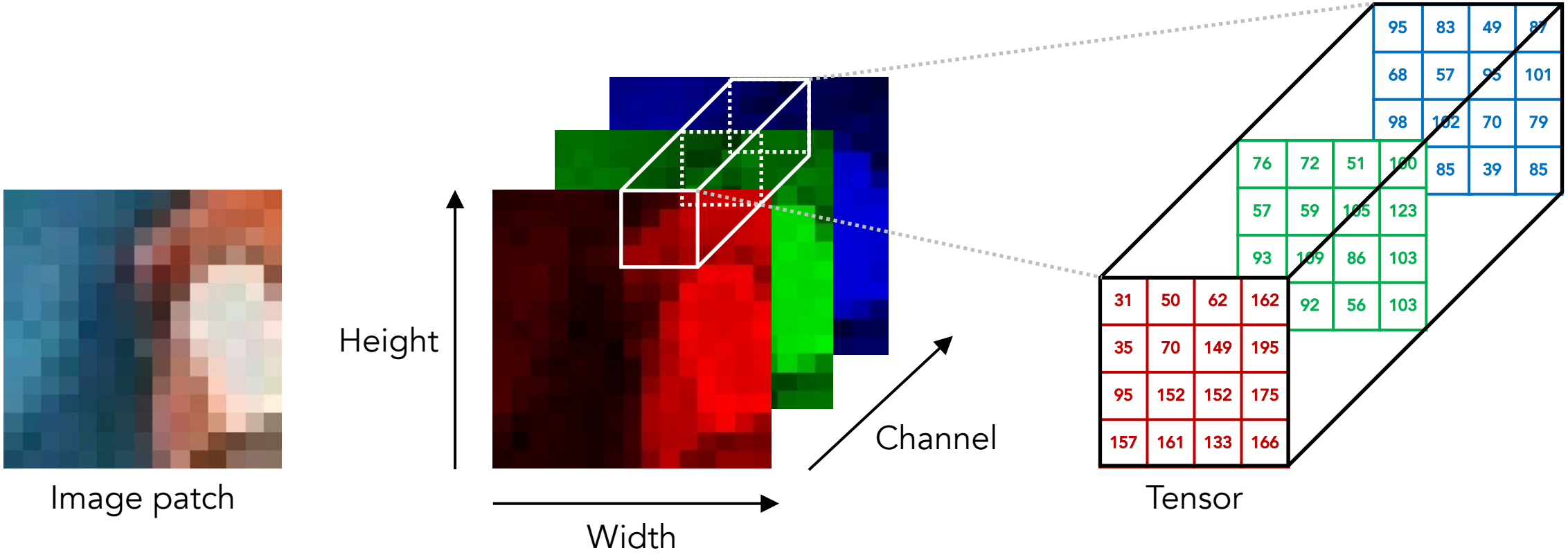

Source -- http://cs231n.stanford.edu/slides/2022/lecture_1_2_ruohan.pdf

People can see images directly, but computers read only numbers. Typically, computers store images as pixels with RGB channels with values ranging from 0 to 255.

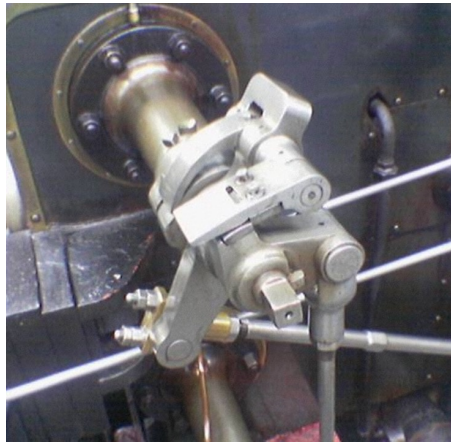



Source -- http://16385.courses.cs.cmu.edu/fall2022/lecture/filtering

Images can be represented as a 3D tensor (width*height*channels). For RGB images, we have 3 channels corresponding to the pixel intensity of red, green, and blue colors.



Before the deep learning era, Computer Vision used hand-crafted features. Researchers developed different image filters/kernels to extract features using convolution.



Input image

Discrete convolution: sum of the element-wise multiplication

Input

| | | | | |
|---|---|---|---|---|
| 7 | 2 | 3 | 3 | 8 |
| 4 | 5 | 3 | 8 | 4 |
| 3 | 3 | 2 | 8 | 4 |
| 2 | 8 | 7 | 2 | 7 |
| 5 | 4 | 4 | 5 | 4 |

*

Filter (Kernel)

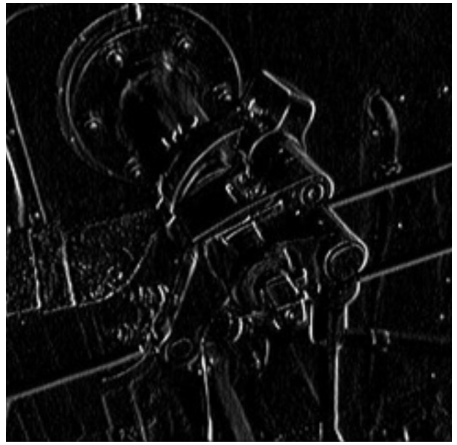
| | | |
|---|---|---|
| 1 | 0 | -1 |
| 1 | 0 | -1 |
| 1 | 0 | -1 |

7x1+4x1+3x1+
2x0+5x0+3x0+
3x-1+3x-1+2x-1
= 6

=

Output

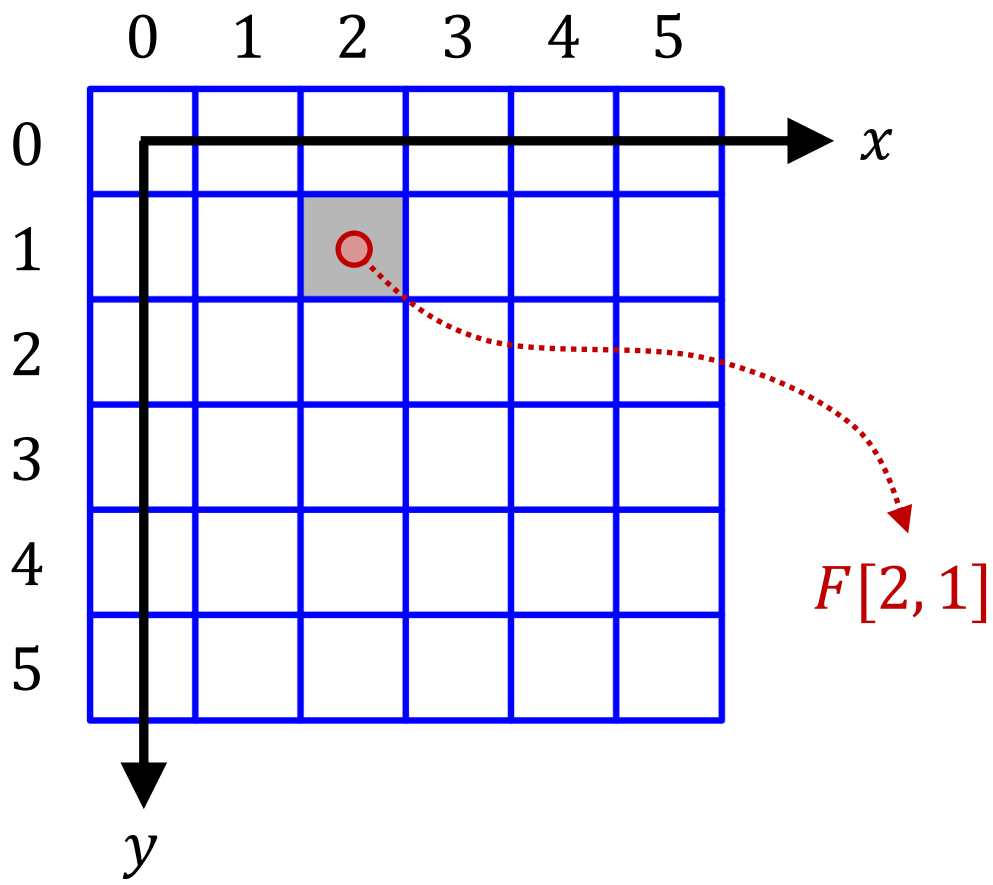
| | | |
|---|---|---|
| 6 | | |
| | | |
| | | |



Output image

Source -- https://docs.opencv.org/4.x/dc/ddf/tutorial_how_to_use_OpenCV_parallel_for_new.html

About image coordinates, the origin is at the top-left pixel. Notation $F[x, y]$ means the value of the center of the pixel for image $F$ at location $[x, y]$ in the 2D array.




$$F[x, y]$$

$F$ is an array of numbers (pixels)

$x, y$ are integers (column/row indices)

Source -- https://courses.cs.washington.edu/courses/cse576/20sp/calendar/

# Let’s run the box filter

$g[0,0]$

$g[\cdot,\cdot]$

kernel

$\frac{1}{9}$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

image $f[\cdot,\cdot]$ $f[1,1]$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

output $h[\cdot,\cdot]$ $h[1,1]$

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note that we assume that the kernel coordinates are centered

$$h[m,n] = \sum_{k,l} g[k,l]\, f[m+k, n+l]$$

output filter image (signal)

Source -- http://16385.courses.cs.cmu.edu/fall2022/lecture/filtering/

# Let’s run the box filter

$g[\cdot,\cdot]$

kernel

$\frac{1}{9}$

| | | |
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| 1 | 1 | 1 |
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image $f[\cdot,\cdot]$

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| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
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output $h[\cdot,\cdot]$

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$$h[m,n] = \sum_{k,l} g[k,l]\, f[m+k, n+l]$$

output filter image (signal)

Source -- http://16385.courses.cs.cmu.edu/fall2022/lecture/filtering/

# Let’s run the box filter

$g[\cdot,\cdot]$

kernel

$\frac{1}{9}$

| | | |
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| 1 | 1 | 1 |
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image $f[\cdot,\cdot]$

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| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
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| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
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output $h[\cdot,\cdot]$

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shift-invariant: as the pixel shifts, so does the kernel

$$h[m,n] = \sum_{k,l} g[k,l]\, f[m+k, n+l]$$

output filter image (signal)

Source -- http://16385.courses.cs.cmu.edu/fall2022/lecture/filtering/

# Let’s run the box filter

$g[\cdot,\cdot]$

kernel

$\frac{1}{9}$

| | | |
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| 1 | 1 | 1 |
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image $f[\cdot,\cdot]$

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| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
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output $h[\cdot,\cdot]$

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$$h[m,n] = \sum_{k,l} g[k,l]\, f[m+k, n+l]$$

output filter image (signal)

Source -- http://16385.courses.cs.cmu.edu/fall2022/lecture/filtering/

# Let’s run the box filter

$g[\cdot,\cdot]$

kernel

$\frac{1}{9}$

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image $f[\cdot,\cdot]$

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| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
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| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
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output $h[\cdot,\cdot]$

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$$h[m,n] = \sum_{k,l} g[k,l]\, f[m+k, n+l]$$

output filter image (signal)

Source -- http://16385.courses.cs.cmu.edu/fall2022/lecture/filtering/

# Let’s run the box filter

$g[\cdot,\cdot]$

kernel

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| 1 | 1 | 1 |
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image $f[\cdot,\cdot]$

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output $h[\cdot,\cdot]$

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$$h[m,n] = \sum_{k,l} g[k,l]\, f[m+k, n+l]$$

output filter image (signal)

Source -- http://16385.courses.cs.cmu.edu/fall2022/lecture/filtering/

# Let’s run the box filter

$g[\cdot,\cdot]$

kernel

$\frac{1}{9}$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

image $f[\cdot,\cdot]$

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|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

output $h[\cdot,\cdot]$

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$$h[m,n] = \sum_{k,l} g[k,l]\, f[m+k, n+l]$$

output filter image (signal)

Source -- http://16385.courses.cs.cmu.edu/fall2022/lecture/filtering/

# Let’s run the box filter

$g[\cdot,\cdot]$

kernel

$\frac{1}{9}$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

image $f[\cdot,\cdot]$

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| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

output $h[\cdot,\cdot]$

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$$h[m,n] = \sum_{k,l} g[k,l]\, f[m+k, n+l]$$

output filter image (signal)

Source -- http://16385.courses.cs.cmu.edu/fall2022/lecture/filtering/

# Let’s run the box filter

$g[\cdot,\cdot]$

kernel

$\frac{1}{9}$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

image $f[\cdot,\cdot]$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
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output $h[\cdot,\cdot]$

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$$h[m,n] = \sum_{k,l} g[k,l]\, f[m+k, n+l]$$

output filter image (signal)

Source -- http://16385.courses.cs.cmu.edu/fall2022/lecture/filtering/

# Let’s run the box filter

$g[\cdot,\cdot]$

kernel

$\frac{1}{9}$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

image $f[\cdot,\cdot]$

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| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
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output $h[\cdot,\cdot]$

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$$h[m,n] = \sum_{k,l} g[k,l]\, f[m+k, n+l]$$

output filter image (signal)

Source -- http://16385.courses.cs.cmu.edu/fall2022/lecture/filtering/

# Let’s run the box filter

$g[\cdot,\cdot]$

kernel

$\frac{1}{9}$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

image $f[\cdot,\cdot]$

| | | | | | | | | | |
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| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

output $h[\cdot,\cdot]$

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| | 0 | 20 | 40 | | | | | | |
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$$h[m,n] = \sum_{k,l} g[k,l]\, f[m+k, n+l]$$

output filter image (signal)

Source -- http://16385.courses.cs.cmu.edu/fall2022/lecture/filtering/

# Let’s run the box filter

$g[\cdot,\cdot]$

kernel

$\frac{1}{9}$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

image $f[\cdot,\cdot]$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

output $h[\cdot,\cdot]$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| | 0 | 10 | 20 | 30 | 30 | 30 | 20 | 10 | |
| | 0 | 20 | 40 | 60 | 60 | 60 | 40 | 20 | |
| | 0 | 30 | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |

$$h[m,n] = \sum_{k,l} g[k,l]\, f[m+k, n+l]$$

output filter image (signal)

Source -- http://16385.courses.cs.cmu.edu/fall2022/lecture/filtering/

# Let’s run the box filter

$g[\cdot,\cdot]$

kernel

$\frac{1}{9}$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

image $f[\cdot,\cdot]$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

output $h[\cdot,\cdot]$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| | 0 | 10 | 20 | 30 | 30 | 30 | 20 | 10 | |
| | 0 | 20 | 40 | 60 | 60 | 60 | 40 | 20 | |
| | 0 | 30 | 50 | 80 | 80 | 90 | 60 | 30 | |
| | 0 | 30 | 50 | 80 | 80 | 90 | 60 | 30 | |
| | 0 | 20 | 30 | 50 | 50 | 60 | 40 | 20 | |
| | 0 | 10 | 20 | 30 | 30 | 30 | 20 | 10 | |
| | 10 | 10 | 10 | 10 | 0 | 0 | 0 | 0 | |
| | 10 | | | | | | | | |
| | | | | | | | | | |

$$h[m,n] = \sum_{k,l} g[k,l]\, f[m+k, n+l]$$

output filter image (signal)

Source -- http://16385.courses.cs.cmu.edu/fall2022/lecture/filtering/

# Let’s run the box filter

$g[\cdot,\cdot]$

kernel

$\frac{1}{9}$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

image $f[\cdot,\cdot]$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

output $h[\cdot,\cdot]$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| | 0 | 10 | 20 | 30 | 30 | 30 | 20 | 10 | |
| | 0 | 20 | 40 | 60 | 60 | 60 | 40 | 20 | |
| | 0 | 30 | 50 | 80 | 80 | 90 | 60 | 30 | |
| | 0 | 30 | 50 | 80 | 80 | 90 | 60 | 30 | |
| | 0 | 20 | 30 | 50 | 50 | 60 | 40 | 20 | |
| | 0 | 10 | 20 | 30 | 30 | 30 | 20 | 10 | |
| | 10 | 10 | 10 | 10 | 0 | 0 | 0 | 0 | |
| | 10 | 10 | 10 | 10 | 0 | 0 | 0 | 0 | |
| | | | | | | | | | |

$$\underbrace{h[m,n]}_{\text{output}} = \sum_{k,l} \underbrace{g[k,l]}_{\text{filter}} \underbrace{f[m+k,n+l]}_{\text{image (signal)}}$$

Source -- http://16385.courses.cs.cmu.edu/fall2022/lecture/filtering/

# … and the result is

$g[\cdot,\cdot]$

kernel

$\frac{1}{9}$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

image $f[\cdot,\cdot]$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

output $h[\cdot,\cdot]$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| | 0 | 10 | 20 | 30 | 30 | 30 | 20 | 10 | |
| | 0 | 20 | 40 | 60 | 60 | 60 | 40 | 20 | |
| | 0 | 30 | 50 | 80 | 80 | 90 | 60 | 30 | |
| | 0 | 30 | 50 | 80 | 80 | 90 | 60 | 30 | |
| | 0 | 20 | 30 | 50 | 50 | 60 | 40 | 20 | |
| | 0 | 10 | 20 | 30 | 30 | 30 | 20 | 10 | |
| | 10 | 10 | 10 | 10 | 0 | 0 | 0 | 0 | |
| | 10 | 10 | 10 | 10 | 0 | 0 | 0 | 0 | |
| | | | | | | | | | |

$$\underbrace{h[m,n]}_{\text{output}} = \sum_{k,l} \underbrace{g[k,l]}_{\text{filter}} \underbrace{f[m+k,n+l]}_{\text{image (signal)}}$$

Source -- http://16385.courses.cs.cmu.edu/fall2022/lecture/filtering/

Exercise 9.1: Compute the result of the following convolution operations (with no padding, stride 1).

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 2 | 0 |
| 0 | 3 | 4 | 0 |
| 0 | 0 | 0 | 0 |

Image

$*$

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 0 |
| 0 | 0 | 0 |

Kernel A

$=?$

Hint:

| | |
|---|---|
| | |
| | 3 |

$*$

| | |
|---|---|
| 1 | -1 |
| 1 | -1 |

Kernel B

$=?$

Hint:

| | | |
|---|---|---|
| | | |
| | | |
| | | 4 |

What will be the result of the following convolution operation?

Input image

$*$

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 0 | 0 | 0 |

Filter
(Kernel)

Source -- https://www.cs.cornell.edu/courses/cs5670/2022sp/lectures/lec01_filter_for_web.pdf

The following convolution operation produces an identical image.

Input image

$*$

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 0 | 0 | 0 |

Filter
(Kernel)

$=$

Identical image

Source -- https://www.cs.cornell.edu/courses/cs5670/2022sp/lectures/lec01_filter_for_web.pdf

What will be the result of the following convolution operation?

Input image

$*$

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 0 |
| 0 | 0 | 0 |

Filter
(Kernel)

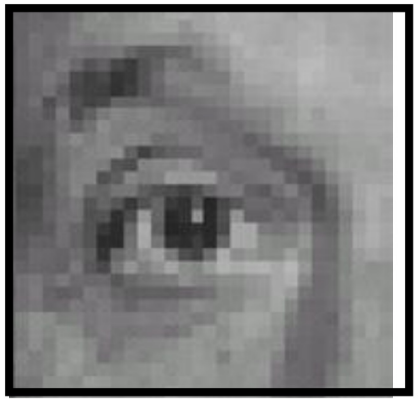
Source -- https://www.cs.cornell.edu/courses/cs5670/2022sp/lectures/lec01_filter_for_web.pdf

The following convolution operation shifts the image left by one pixel.

Input image

$$*\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} =$$

Filter
(Kernel)



Shifted by 1 pixel

Source -- https://www.cs.cornell.edu/courses/cs5670/2022sp/lectures/lec01_filter_for_web.pdf

What will be the result of the following convolution operation?

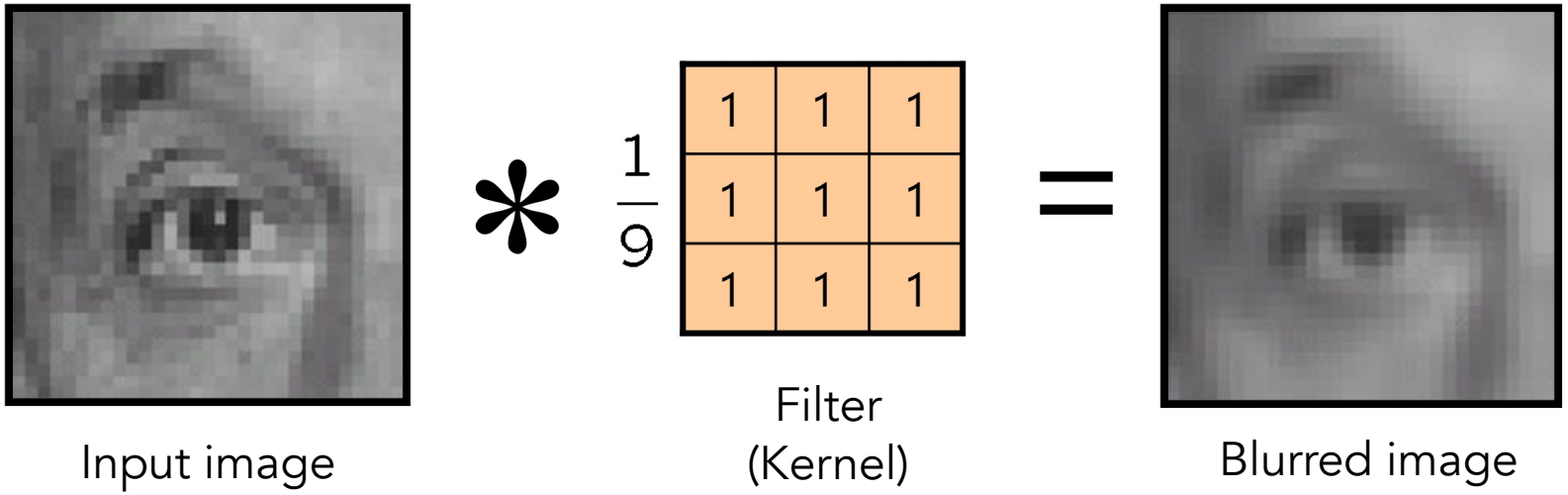
Input image

$$*\ \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Filter
(Kernel)

Source -- https://www.cs.cornell.edu/courses/cs5670/2022sp/lectures/lec01_filter_for_web.pdf

The following convolution operation blurs the image.




Source -- https://www.cs.cornell.edu/courses/cs5670/2022sp/lectures/lec01_filter_for_web.pdf

What will be the result of the following convolution operation?

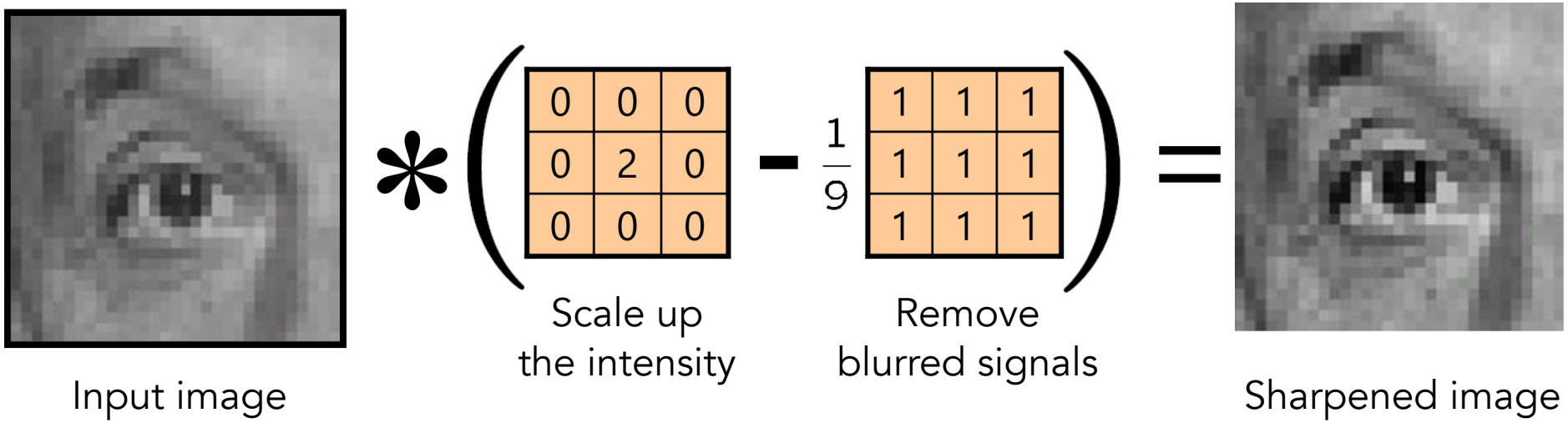
Input image

$$* \left( \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \right)$$

Filter
(Kernel)

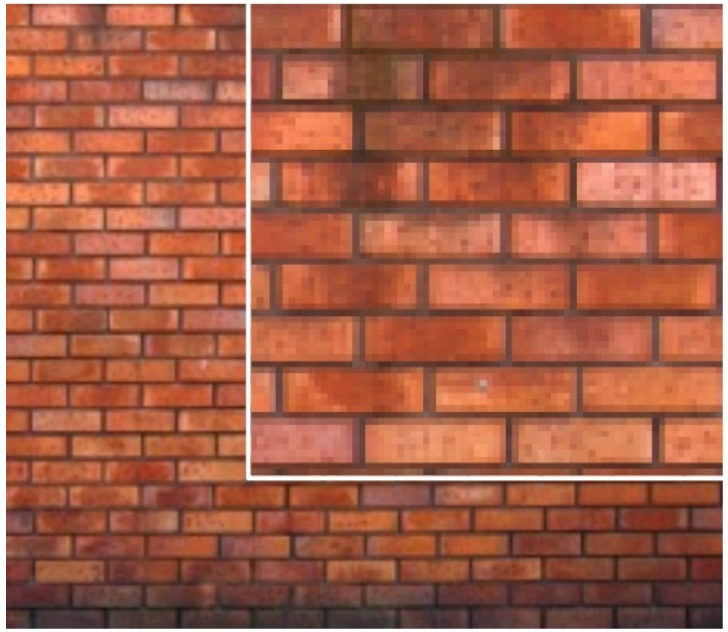
Source -- https://www.cs.cornell.edu/courses/cs5670/2022sp/lectures/lec01_filter_for_web.pdf

The following convolution operation sharpens the image.



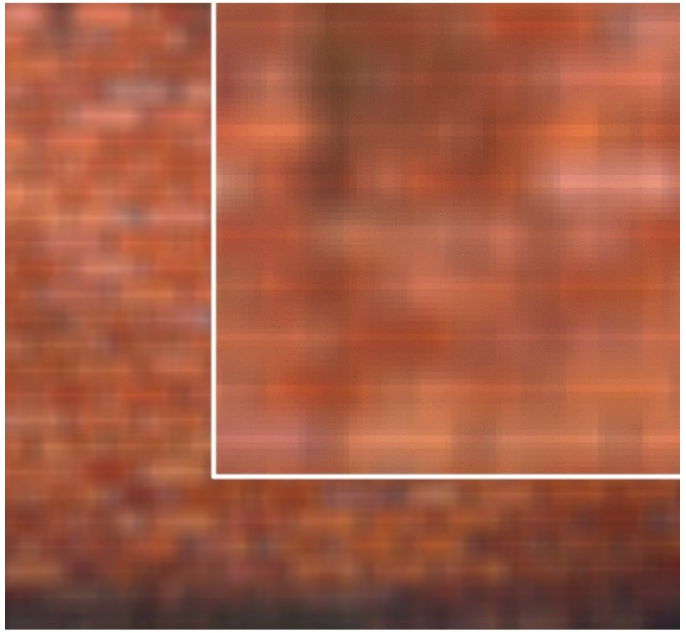

Source -- https://www.cs.cornell.edu/courses/cs5670/2022sp/lectures/lec01_filter_for_web.pdf

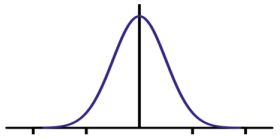
We can use image filters to blur images, such as using the box filter and the Gaussian filter (2D Gaussian distribution).



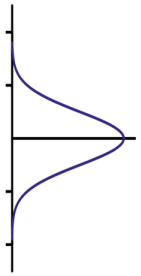
box filter (i.e., mean filter)

$$\frac{1}{9} \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

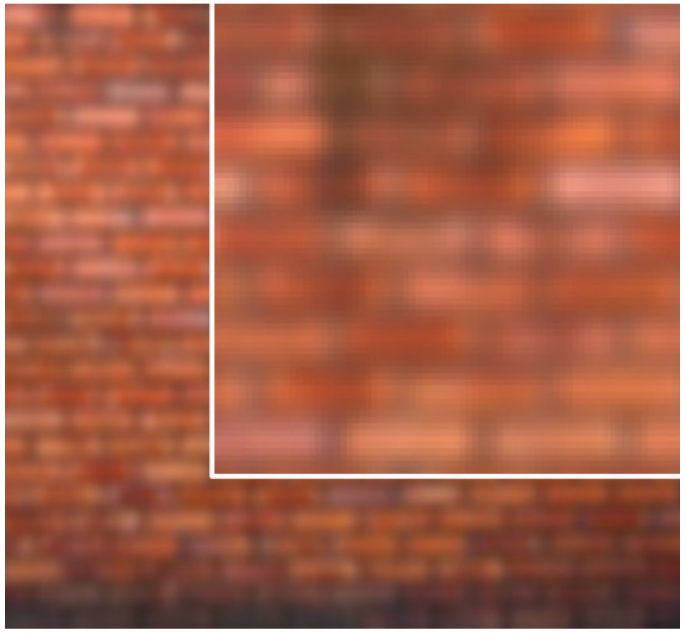




$$\frac{1}{16} \times \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$



Gaussian filter

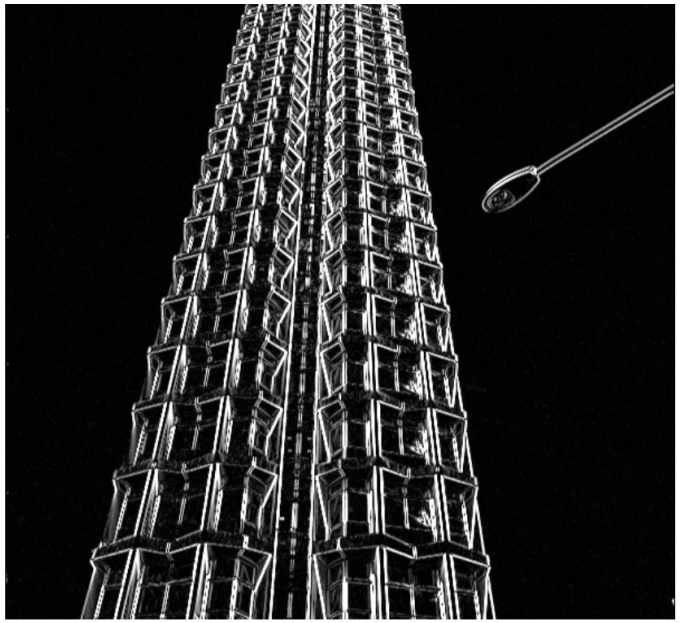


Source -- http://16385.courses.cs.cmu.edu/fall2022/lecture/filtering

We can use image filters to detect edges, such as using the Sobel filter to compute the derivative of signals.

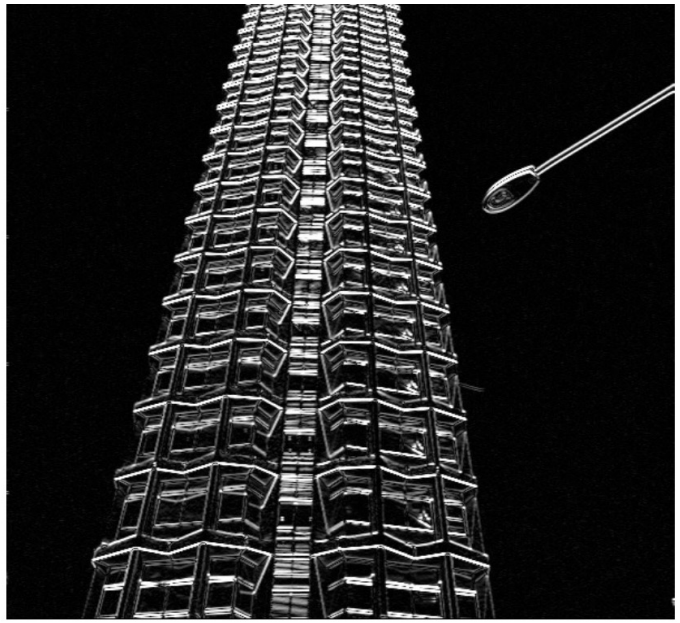
Horizontal Sobel filter

| | | |
|---|---|---|
| 1 | 0 | -1 |
| 2 | 0 | -2 |
| 1 | 0 | -1 |



| | | |
|---|---|---|
| 1 | 2 | 1 |
| 0 | 0 | 0 |
| -1 | -2 | -1 |

Vertical Sobel filter

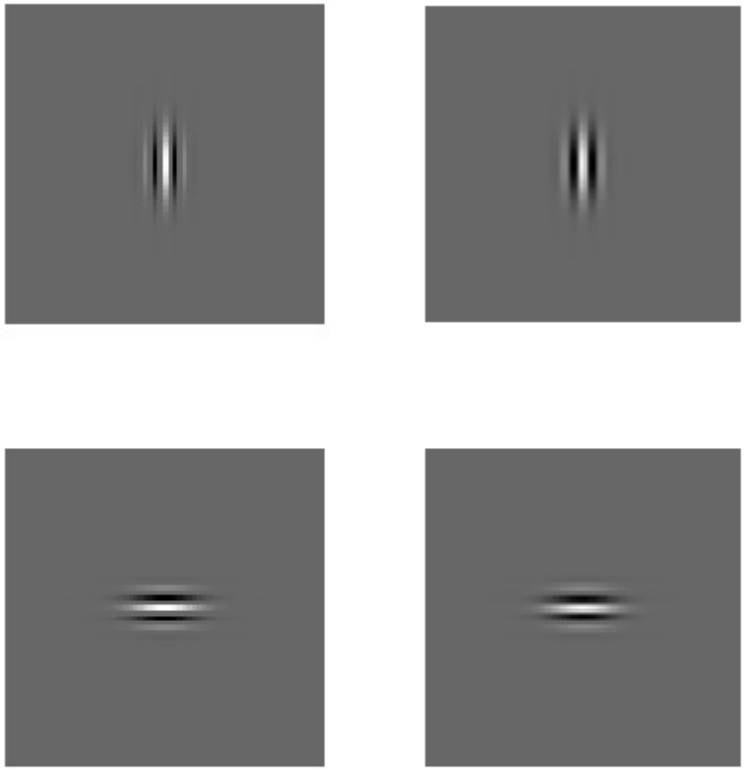


Source -- http://16385.courses.cs.cmu.edu/fall2022/lecture/filtering

We can use various hand-crafted convolutional kernels (i.e., filter banks) to extract different kinds of information in the image, such as using the Gabor filters.

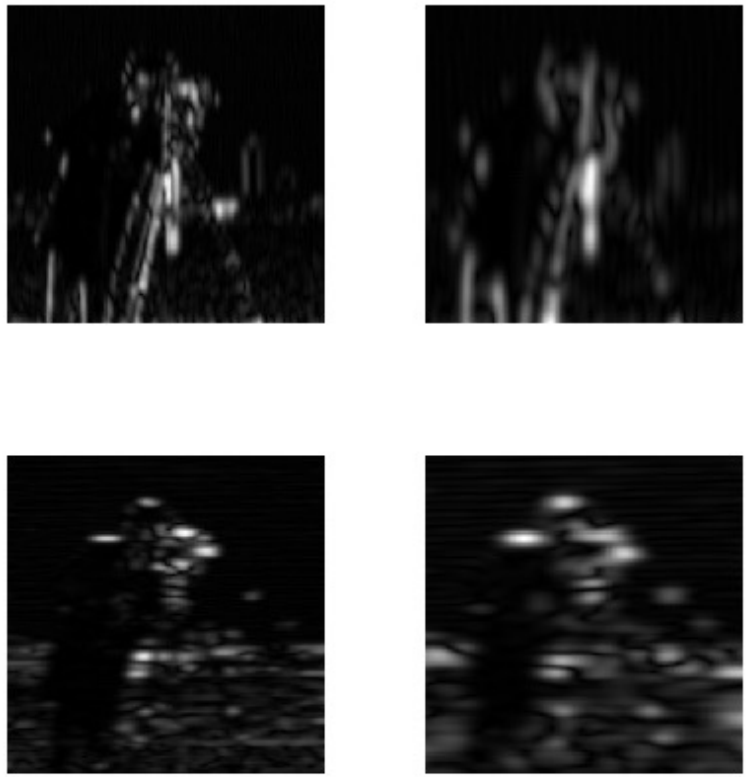
Input image

*



A partial set of Gabor filters

=

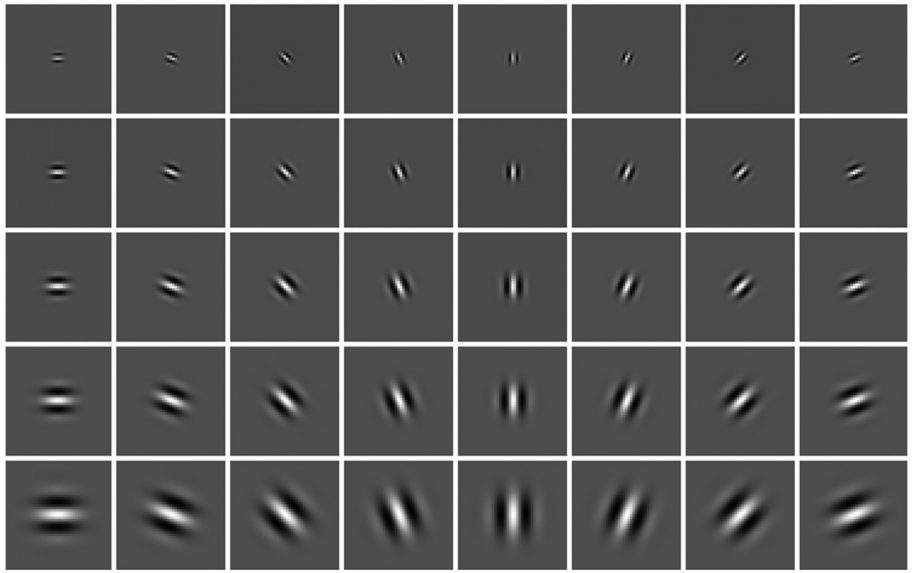


Output images

Source -- https://www.baeldung.com/cs/ml-gabor-filters

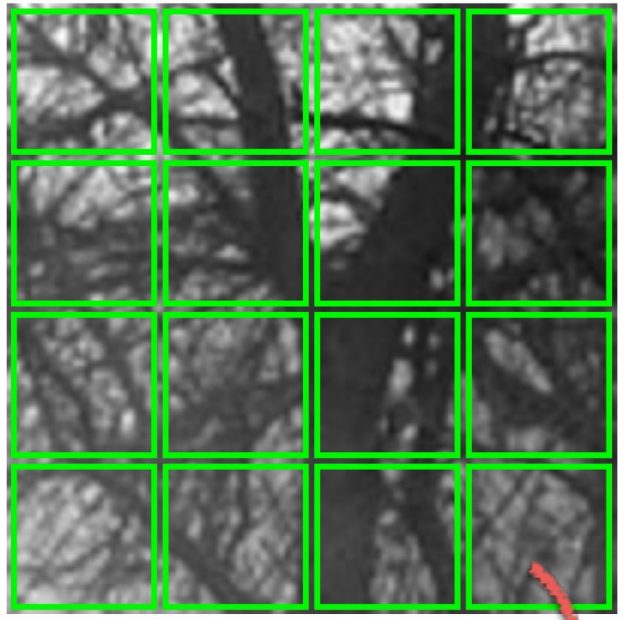
To construct features, we can perform convolution on image patches using a set of kernels (i.e., filter banks) and then aggregate them into a feature vector.

A set of $N$ convolutional kernels



For each cell, compute filter responses using convolution, and then average them

4 x 4 cell

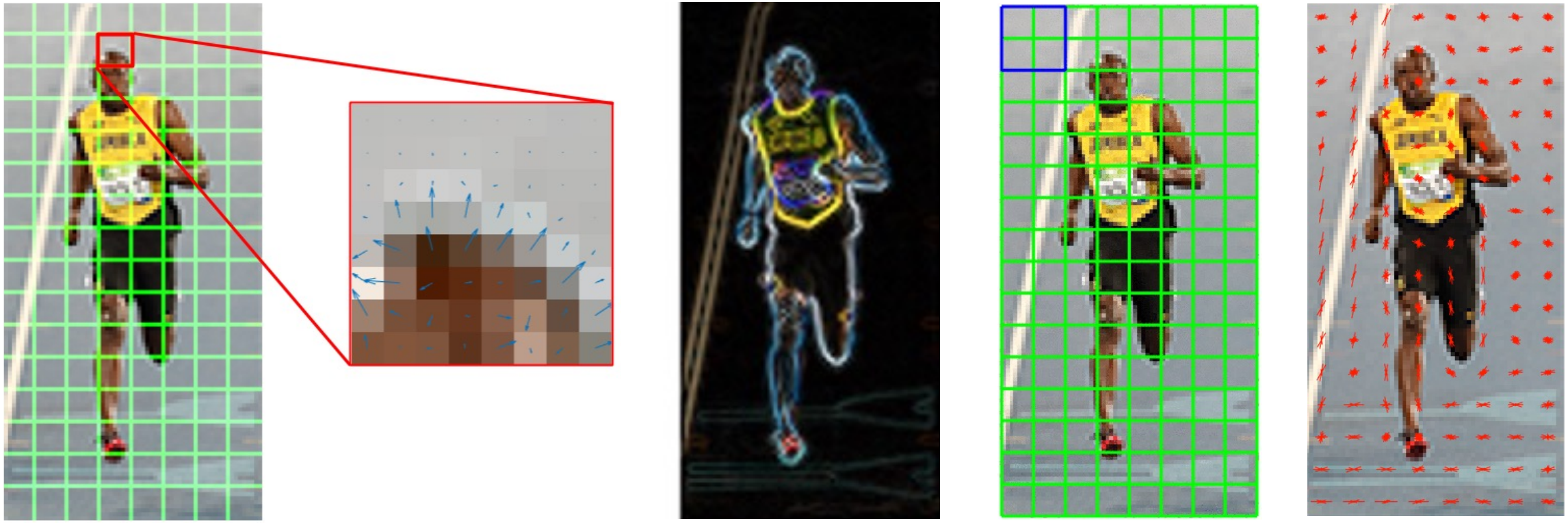


Get a $4 \times 4 \times N$ feature vector

averaged filter responses

Source -- http://16385.courses.cs.cmu.edu/fall2022/lecture/features

There are many ways of constructing feature vectors, such as Histogram of Gradients.



Compute gradients using Sobel filter for each image patch → Aggregate (for each patch) and normalize gradients into histograms

Source -- https://learnopencv.com/histogram-of-oriented-gradients/

# Take-Away Messages

- Computers represent images as tensors with numbers in several channels (e.g., red, green, and blue).
- The origin (i.e., index $[0,0]$) of the image coordinate is at the top-left corner of the image.
- Convolution (the discrete version) means sliding a kernel/filter over the image to compute the sum of element-wise multiplication, which can also be seen as a dot product.
- We can use convolution to perform many different image-filtering operations, such as shifting pixels, blurring/sharpening images, and detecting edges.
- We can use a filter bank (i.e., a collection of kernels) to perform convolution on image patches, and the result (i.e., a set of filter responses) can be used as features.

# Questions?